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41. Mathematical and Astronomical Insights of Ancient Indian Scholars: A Scientific Exploration

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Abstract

This paper explores the profound contributions of ancient India towards mathematics and astronomy, focusing on the intellectual legacy of the Mahaviharas like Nalanda. Over seven centuries, scholars such as Aryabhata, Varahamihira, Brahmagupta, and Bhaskara made groundbreaking advancements in trigonometry, algebra, and astronomy. Their work is examined alongside modern discoveries, particularly the alignment of ancient planetary models with later European theories. The paper also highlights the Ujjain Astro-lab and its role in setting the time meridian. As a further extension, the orbit equation is deduced using the mathematical concepts of ancient scholars, offering a fascinating link between ancient and contemporary understandings of the cosmos and demonstrating the continuity of knowledge from early astronomical insights to modern orbital equations.

1. Introduction

The ancient Mahaviharas, with Nalanda as a prime example, were hubs of intellectual brilliance from their establishment until the 12th century [1-3]. While they were celebrated for their contributions to Buddhist philosophy, these institutions also played a key role in advancing mathematics and astronomy. Over seven centuries, Nalanda drew scholars from diverse regions of Asia, including China, Korea, Japan, and Sri Lanka, promoting a dynamic exchange of ideas [4-5]. Its enduring influence helped to shape the intellectual and scientific development of Asia, leaving a profound mark on global civilizations [6-27]. It is important to note that all the Sanskrit verses with meanings as described in the following sections are takes from ref. [1-27].

In this paper, at first, all the mathematical as well as astronomical developments of ancient Indian scholars will be described in short with proper chronology and scientific justification. More specifically, the contribution to trigonometry by *Aryabhata* (476–550 CE) and *Varahamihira* (505–587 CE), the work of Brahmagupta (598–668 CE) in algebra, the development of Bhaskara I (600–680 CE) and Bhaskara II (1114–1185 CE) will be revisited. After that, we shall specifically focus our discussion on the ancient astronomical findings with a few modern numerical findings. Notably, a brief overview of ancient astronomical instrument along with the justification of Ujjain Astro-lab will also be discussed. In addition, effort will be given to justify the ancient planetary model as a geometrical two-body central force problem in accordance with the precalculus geometrical models of European astronomer.

2. Opulent Legacy of Vedic-Indian Scholars: A Short Review

The Vedic period (c. 1500 BCE - 500 BCE), known for religious texts like the Vedas, also laid the foundation for early mathematical concepts in geometry, arithmetic, and astronomy. Pingala (c. 3rd–2nd century BCE), one of India's earliest mathematicians, made significant contributions to combinatorics and the development of the binary number system in his work '*Chandahsutra'*, predating concepts like the Fibonacci sequence and binomial coefficients. While Sushruta (c. 6th century BCE) is renowned for his work in medicine, his contributions also involved practical mathematics, particularly in measurement and geometry. Ancient Indian mathematics, especially in arithmetic, algebra, and geometry, set the stage for later developments. It is also important to note that, the exact wording of the sutra and verse no. may vary slightly depending on the edition. But, in the verse,

"गुरुर्लघुश्च द्वौ च तौ च यत्र तच्छन्दः।" ($Chandahsutra$ 1.2)

(*Gurur laghuśca dvau ca tau ca yatra tacchandaḥ.*)

"Guruḥ laghuśca" (A heavy (guru) syllable and a light (laghu) syllable).
"Dvau ca tau ca" (These two (syllables) and their combinations.)
"Yatra tacchandaḥ" [Wherever these (combinations) occur, that is chandaḥ (prosody/meter)].

he outlines his basic idea of binary choices in meter arrangement. He introduces Pingala introduces two types of syllables. One is '**Short syllables** (−)' are denoted by **0** and the another is '**Long syllables** (|), are denoted by **1**. Thus, for a 3-syllable meter, one can easily obtain all possible combinations of short and long syllables as binary numbers like, **000**: (− − −), **001**: (− − |) and so on. Also, through another verse he indicates for 3-syllable meter, the number of combinations is $2^3 = 8$. Similarly for a For example, with a 4-syllable sequence, valid combinations (where no two long syllables are adjacent) are $\mathbf{000}$: $(- - - -)$, $\mathbf{0010}$: $(- - | -)$, $\mathbf{0100}$: $(- | - -)$, $\mathbf{1000}$: $(| - - -)$**.** In addition, He also addresses the problems where no two long syllables (|) are adjacent. This type of counting is similar to the 'Fibonacci sequence', where each term is the sum of the previous two that result the recurrence $f(n) = f(n-1) + f(n-2)$, where $f(1) = 2, f(2) = 3$, and so on.

Though Pingala does not introduce Pascal's Triangle directly but his verse demonstrates the connection to Pascal's Triangle and binomial coefficients. We can say, Pingala would have considered how many ways we can arrange long syllables (|) in a fixed-length sequence. As For example, if we consider his verse with 4 syllables, the number of ways to place l long syllables (|) in a sequence of 4 positions is given by the binomial coefficient (4 $choose$ l). Hence, the binomial coefficients are, $(4\ choose\ 0) = 1, (4\ choose\ 1) = 4, (4\ choose\ 2) = 6, (4\ choose\ 3) =$

$4, (4\ choose\ 1) = 4$, where all these values correspond to the number of ways to arrange 0, 1, 2, 3, and 4 long syllables in a sequence of 4 syllables, and they match the entries in the 4th row of Pascal's Triangle.

Sushruta (c. 6th century BCE), author of the *Sushruta Samhita* and a foundational figure in Ayurveda, is primarily known for his contributions to medicine and surgery. While his work involved practical applications of measurement and geometry, which intersect with mathematics, this article focuses on astro-mathematical developments, so his contributions are not explored further here.

2.1 The Idea of Cosmos: Ancient Insights into Astro-mathematics (476–550 CE)

The early contributors about the idea of cosmos are Aryabhata and Varahamihira. Aryabhata (c. 476–550 CE) is believed to have been born in Kerala, though there are debates about his exact birthplace. His early life or teachers are not unveiled properly. It is believed that he was greatly influenced by the mathematical and astronomical traditions of Brahmagupta. But, some scholars suggest that he might have been studied at the famous Nalanda Mahavihara. He is one of the most brilliant minds in ancient India wrote Aryabhatiya in which the foundation of Astro-mathematics was described with deep insight about the concept of modern-day cosmos. In 'Aryabhatiya', total no. of verses are 118, and it is subdivided in for chapters, namely, Ganita Paddhati (33 verses), Kala Paddhati (25 verses), Gola Paddhati (24 verses), and Siddhanta Paddhati (36). Here, in this section we shall display some key 'Sanskrit verses' from 'Aryabhatiya' offering an overview of his legacy towards astronomy and mathematics (as coined as Astro-mathematics).

The revolutionary idea was that the 'Earth rotates on its axis', was not widely accepted in the West as it was against the concept of bible. In his verses, he also described the 'apparent motion of the Sun' as a result of the Earth's rotation and conceptualize the Model of the Solar System. Another revolutionary idea of him was that the 'Earth rotates on its axis' which was not widely accepted in the West as it was against the concept of bible. He also described the 'apparent motion of the Sun' as a result of the Earth's rotation. In another verse the Model of the Solar System was properly described. In addition, Aryabhata also gave the basic concept of trigonometry, especially the sine (called "Jya" in Sanskrit) functions a long before the West by calculating the length of shadows and the angles of the Sun. The calculation of time as Aryabhata defined was the precise estimates for the length of the 'solar year' and the 'sidereal day' (the time taken by the Earth to rotate once with respect to the stars, other than the Sun). The time span of day and year was also calculated by Aryabhata which are nearly close to the modern accepted values. As discussed earlier, the verse '*Aryabhatiya* 1.9' tells Aryabhata's view that the 'Earth rotates on its axis'. It was the 'heliocentric idea' that described in Nalanda a few centuries before the proposed model of Copernicus. It is also important to note that Aryabhata tried to calculate the 'Earth's circumference' and its 'diameter', depending on his knowledge of the celestial sphere and the length of the day. Frankly speaking, his calculations were not perfectly accurate by modern standards, but is too close

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to the actual values. Though west always criticizes India about the ignorance of Indian people towards eclipses, it is quite surprising that, Aryabhata, long before the British rule, had made significant contributions to understand the motion of the Moon along with its phases, incorporating both the lunar cycle and planetary motion in his astronomical framework. He also describes lunar and solar eclipses with a proper explanation of the shadow of the Earth or the Moon. It is also to be noted that Aryabhata was one of the earliest mathematicians (there are some contradictions about the first one) to use the concept of zero (or the "shunya" in Sanskrit). This central concept was very essential in mathematics as well as in astronomical calculations to find out the details about the positions of planets, orbits, and the details of eclipses. He also calculated the value of π (Pi) as 3.1416. This value is close to the modern value of 3.14159.

'Varahamihira', a key figure in ancient Indian astronomy and mathematics, made foundational contributions that shaped the astro-mathematical framework. His works '*Pancha-siddhantika'* and *Brihat Samhita* (the former now lost, with only Utpala's commentary remaining) offer detailed insights into mathematical and astronomical concepts. He also used arithmetic progressions (AP) to calculate planetary positions over time and applied the decimal place value system, though he did not explicitly work with zero like Brahmagupta. His verses also discuss periodic cycles, including lunar months and solar years, and introduce the idea of cyclical time, such as 'kalpas' (eons). Notably, Varahamihira's understanding of geometry included the sine function, which was essential for his astronomical calculations, and he also formulated the Pythagorean theorem long before Pythagoras.

In addition to his work on astronomy, Varahamihira's texts demonstrate the division and multiplication of large numbers, crucial for calculating celestial parameters. He even provided a formula for the area of a circle to model the motion of celestial bodies. These contributions highlight his integral role in both mathematics and astronomy.

2.2 Brahmagupta (c. 598–668 CE): The Radiant Trailblazer in Astro-Mathematics

Brahmagupta is one of the first mathematicians to provide a systematic treatment of zero and negative numbers, find out actual method of solving quadratic equation, calculate 'mean longitude' of a planet after a given number of days, by adjusting for the planet's 'angular velocity' and the 'eccentricity' of its orbit. The extensive calculation about the prediction of eclipse, develop trigonometric tables, and develop formula for cyclic quadrilateral area. This cyclic quadrilateral area is very important for astronomical mapping.

2.3 Bhaskara I (c. 600–680 CE) and Bhaskara II (c. 1114–1185 CE): Precursor of Next-Gen Developments

Bhaskara I (c. 600–680 CE) is best known for his commentaries and expansion on Aryabhata's work. By using 'jya' and 'kojya' he expanded the Aryabhata's work up to a new height. He worked out

'Diophantine equations' using geometric method. Bhaskara II (c. 1114–1185 CE) was also an renowned mathematician and astronomer for his groundbreaking contributions to algebra, calculus, trigonometry, and astronomy. His notable works, including Lilavati, Bijaganita, and Siddhanta Shiromani, influenced both the development of Indian and global science. He also introduces the concept of rudimentary calculus before Leibnitz and Newton.

2.4 Other Notable Contributions:

Madhava of Sangamagrama ((c. 1340–1425 CE) is one of the founders of the Kerala School of Astronomy and Mathematics, a group of scholars who were ahead of their time and had an important impact on both Indian and global scientific thought. Madhava's contributions to infinite series are regarded as some of his most important achievements. His groundbreaking work in this area formed the basis for the eventual development of calculus in Europe. He also developed an infinite series for the calculation of π (pi) as,

$$\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots\right) \tag{1}$$

Madhava also develop trigonometric series, which were precursors to the 'Taylor series' and 'Maclaurin series developed in Europe in the 17th century. Madhava's contributions to astronomy are also highly appreciated. He explored the boundaries of heliocentric model extensively for the solar system, which were later elaborated upon by Copernicus. He is credited for introducing differential methods in a rudimentary form to find out the positions of celestial bodies. Additionally, his contributions to the 'sine function' and 'trigonometric tables' played an important role in astronomical calculations.

Nilakantha, another talented figure in the Kerala School, explored the work of Madhava. He also contributed basically on trigonometry, 'approximations of π'. The value of 'Pi' as he introduced is known as 'Nilkantha series' as,

$$\pi = 3 + 4\left(\frac{1}{2 \cdot 3 \cdot 4} - \frac{1}{4 \cdot 5 \cdot 6} + \frac{1}{6 \cdot 7 \cdot 8} - \cdots\right) \tag{2}$$

However, his 'mathematical and astronomical models' were often used by later scholars like Jyesthadeva and Raghunatha to further refine and extend the concepts of trigonometry, calculus, and astronomy.

2.5. Timekeeping Tradition of Ujjain Astronomical Observatory

One might wonder how such advanced astro-mathematical developments were possible without detailed observations. Varahamihira, in his *Brihat Samhita* (6th century CE), references instruments like gnomons, quadrants, and time-measuring devices used for astronomical calculations.

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Additionally, sources suggest that Nalanda had an observatory tower, further supporting the idea of extensive astronomical observations in ancient India. Ujjain, another key centre, was renowned for its astro-lab, where precise timekeeping and celestial measurements were conducted, with the help of accurate astronomical models. It is also important to note that if we shift the 'Time Meridian' (see Figure 1)in Ujjain the accuracy of standard time for India will be more useful and convenient.

3. Derivation of 'Equation of Orbit' using concepts of Ancient Indian Geometry

Looking into the translation of Ptolemy's *Almagest*, Book 7, Chapter 2, one can see, he is citing the Indian astro-mathematical legacy as *"The Indians, in the course of their investigations, have discovered a method for determining the lengths of the years of the planets, a method that appears superior to that which we use in our own system of astronomy."* . But similarly, according to Toomer's translation, 1984, Ptolemy's *Almagest says "We shall now set out the mean motions of the planets... derived from the more accurate observations made by ourselves and our predecessors."*. However, it will become mathematically true if one derives the well-known orbit equation using the mathematical concept of Ancient Indian text [25-28]. In order to derive it, let us first draw an ellipse as shown in Figure 2. Now, we see from the earlier section that Bhramhagupta has the idea of conic section, and or ellipse. Now, if we design the length of major axis, minor axis, and ff' are a, b, and $2c$ respectively, we can easily write from the diagram that,

$$r = 2c\ Cos\varphi + r' cos2\theta\ , \tag{3}$$

From the law of cosine (which was also worked out in ancient text), we can say easily, for triangle fPf'

$$r'^2 = (2a - r)^2 = r^2 - 4cr\ cos\varphi + 2c^2, \tag{4}$$

Then, setting $b^2 = a^2 - c^2$one can obtain the equation of a conic as,

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Figure 1 : The above map shows the numeric affects (See the two bar below) if the 'Time Meridian' set at Ujjain. The modified map as given below shows the modified world map after setting the 'Time Meridian' at Ujjain

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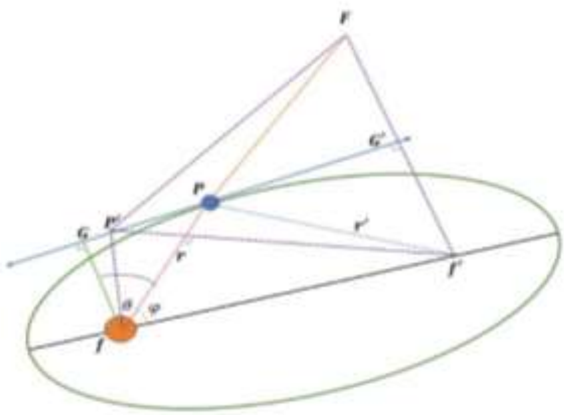

Figure 2: The elliptic orbit of a planet (Blue sphere) with Sun (yellow Sphere)

$$\frac{b^2}{r} = a - c\, cos\varphi. \tag{5}$$

This is the very familiar equation of orbit as deduced using Newton's and Leibnitz calculus.

4. Equivalence between 'Keplerian planetary- Model' and 'Aryabhata-Bhaskara Planetary Model'

Bhaskara I (a scholar of Aryabhata school) in his 'Aryabhatiya-bhasya' assumed the radius of the 'manda' epicycle varies in the same way as the 'karna'. Thus, for the epicyclic and eccentric models of planetary motion, one can arrived at,

$$R\, sin(\Delta\varphi) = \frac{r_0}{R} Rsin(\,\varphi_0 - \varphi_m). \tag{6}$$

Where, R is the Radius of the deferent circle, $\Delta\varphi = \varphi_0 - \varphi_{MS}$, r_0 is the mean value of the 'manda' epicycle. φ_0, φ_m, φ_{MS} are 'mean longitude of the planet', 'manda kon' and 'manda sphuta' respectively. It is important to note that the predicted vale of $\frac{r_0}{R}$ is comparable to $2e$, where e is related to the eccentricity.Now, using Kepler's law, and setting the Sun to Aries line as one of the references, one can even obtain,

$$\varphi_h - \omega t = \varphi_h - \varphi_0 = -2e\sin(\,\varphi_0 - \varphi_a) + \mathcal{A}(e^2)\,. \tag{7}$$

Where, ω is the mean angular velocity of the planet, φ_h , φ_0 are the actual heliocentric longitude and mean longitude of the planet, and $\mathcal{A}(e^2)$ is the anomaly. It is also important to be noted that, $(\,\varphi_0 - \varphi_a)$, is the difference between the longitudes of the mean planet and the apogee. This difference is known as 'anomally' and eventually it can be named as 'manda-kendra' in Indian astronomy.

5. Comparative Analysis

A comparative analysis for different tradition with proper details is displayed in 'Table 1".

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Aspect	**Indian Astronomical Tradition**	**Islamic Astronomical Tradition**	**European Astronomical Tradition**
Mathematical System **Ref. [10-12]**	Decimal system, concept of zero, trigonometric functions (sine, cosine). *(5th century BCE)*	Adopted Indian methods, expanded trigonometry, algebra. *(8th–15th century CE)*	European adoption of Indian numerals, developed trigonometry. *(12th–17th century CE)*

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Aspect	**Indian Astronomical Tradition**	**Islamic Astronomical Tradition**	**European Astronomical Tradition**
Planetary Motion Models **Ref.[13-15]**	Epicycles, planetary models with eccentricity and cycles *(5th century BCE)*	Refinement of Indian models, improved planetary motion tables. *(8th–14th century CE)*	Influence from Islamic astronomy, Copernican revolution. *(16th century CE)*
Astronomical Instruments **Ref.[16-18]**	Early use of astrolabes and quadrants. *(1st millennium BCE)*	Refinement of Indian instruments like astrolabes. *(8th–14th century CE)*	Adaptation of Islamic instruments, increased precision. *(16th century CE)*
Transmission of Knowledge **Ref.[19-21]**	Texts like Surya Siddhanta, Aryabhatiya. *(4th century CE)*	Translations of Indian works, key role of House of Wisdom *(8th–14th century CE)*	Translations of Islamic works during the 12th century *(12th century CE)*
Influence on Key Scholars **Ref.[22-24]**	Aryabhata, Brahmagupta, Bhaskara. *(5th–12th century CE)*	Al-Battani, Al-Khwarizmi, Ibn Sina (Avicenna)' *(5th–12th century CE)*	Copernicus, Tycho Brahe, Johannes Kepler. *(16th–17th century CE)*
Sidereal Year **Ref.[25-27]**	Precise calculation of the sidereal year (365.25636 days). *(5th century BCE)*	Adoption and refinement of Indian calculations. *(8th–14th century CE)*	Refined in European astronomy through Islamic texts. *(16th century CE)*

Table 1: The chronological details of astro-mathematical developments of different traditions.

6. Conclusion

In conclusion, the ancient Indian knowledge system, particularly in mathematics and astronomy, laid the foundation for many scientific principles. This paper highlights the alignment of ancient Indian planetary models with the modern Keplerian model. Notably, an orbit equation is derived using ancient geometric concepts, reflecting the accuracy of these models. Additionally, setting the time meridian at Ujjain proves the set up of world-time error-free by calculating global timelines and offers a potential problem for further exploration.

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Additionally, this work also provides valuable pedagogical examples for interdisciplinary courses in mathematics and astronomy**.**

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